Comparison between GMM and KDE Data Fusion methods for Particle Filtering: Application to Pedestrian Detection from Laser and Video Measurements

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Abstract – *In urban environment, pedestrian detection is a challenging task in automotive research, which often suffers from the lack of reliability due to the occurrences of spurious detections. In order to answer multitarget multisensor tracking problem and more specifically pedestrian tracking, we propose to use an algorithm based on a stochastic recursive Bayesian framework also called particle filter. We aim to solve the problem of consistent Bayesian Decentralized Data Fusion (BDDF) with particle filter using two different statistics approaches in order to better represent the particle set and maintains an accurate summary of the particles. We propose a comparison between a Kernel Density Estimation (KDE) based on non-parametric estimation and a Gaussian Mixture Model (GMM) based on parametric estimation. This approach allows to cope with non-linear models and multi-modalities induced by occlusions and clutters. These two algorithms differ in the representation of particle set during data fusion. Simulation results as well as the results of the experiments conducted on real data demonstrate the relevance of these approaches.*

Keywords: Particle filters, kernel density estimation, Gaussian mixture model, laserscanner, video camera, sensor fusion.

1 Introduction

Multitarget multisensor tracking deals with the state estimation of an unknown number of moving targets. To perform multitarget multisensor tracking the observer can rely on a huge amount of data, possibly collected on multiple receivers.

For a broad review of the various sensors used for pedestrian detection, one can consult [3] where piezoelectric, radar, ultrasound, laser range scanner sensors and cameras operating in the visible or in the infrared are described. Using video sensors to solve the problems of detection and identification seems natural at first, given the capacity of this type of sensor to detect/analyze the size, the shape and the texture of a pedestrian. Many methods to detect human beings were developed in computer vision based on monocular or stereoscopic images [5]. However, the strong sensitivity to atmospheric conditions, the wide variability of human appearance, the limited aperture of this sensor and the impossibility to obtain direct and accurate information concerning depth have, given rise to an interest for a detection method starting using active sensors like a radar or a laser sensor. Indeed the laser system based pedestrian detection has a strong ability in counting and tracking pedestrians, even in the case of a very high-density crowd [6]. However, the obvious limitations of this sensor (no information about shape, contour, texture, color of objects), its sensibility to atmospheric conditions such as rain and fog and the frequent occlusions between objects, require to devise a method of laser/camera fusion to improve a pedestrian collision avoidance system.

A study of sensor-based pedestrian detection, presented in [4], indicates that the laser scanner cooperation with camera is a good solution to be developed. This solution has been chosen in the CityVIP project context (Automated Individual Public Vehicle adapted to urban environment) which aims at improving road safety, mainly focusing safe navigation of smart vehicle [2]. So, the problem is how to combine the diverse and sometimes conflicting amounts of information in the best manner, to outperform the best results expected from the use of a single sensor technology.

The main difficulty of data fusion lies in the association of the new observations coming from different sensors. Thus, two distinct problems have to be jointly solved: data association and the estimation.

The conventional approaches are based on the Kalman filter [7], or its linearized extension [8], and lead to data association algorithms such as the JPDAF [9], the MHT [10] or the PMHT [11] which differ in their association techniques but which all share the same Gaussian assumption. Such algorithms were used to solve many problems resulting from signal or image processing but they regularly failed to work out in case of a non-linear evolution model or in case of non-Gaussian noise models. Over the last years, the Monte Carlo methods became interesting particularly with the particle filtering [12] which establishes its superiority over others for non-linear filtering. The idea consists in representing the law of probability of the states conditioned by the observations by means of a finished weighed sum of Dirac which evolves according to its weights and the object dynamic model, depending on observations. In this paper, two methodologies are described for performing BDDF in order to maintain the stochastic aspect of the particle set. The first method is a transformation of the particle representation to a GMM. The second algorithm approximates the particles by a KDE representation.

The article is organized as follows. In Section 2, the principle of the approach is described and the sensors used by Renault manufacturer are presented. In Section 3, the principles of particle filters are briefly reminded. Section 4 describes the two methods for performing data fusion based on particle filters. In Section 5, simulation and experimental results are presented. Conclusions are proposed in Section 6.

2 Overview

2.1 Our approach

The purpose of this work is to track pedestrians from a moving vehicle. To develop our own approach, our research interest is focused on particle filter [13] because the pedestrian movements are non-linear and non-Gaussian assumption is needed.

Multitarget-multisensor tracking with a particle filter generally uses a data association step, in which each target is mapped to an observation sensor. Conventional approaches propose a Gaussian framework where a covariance matrix [7] is computed from the set. Since the particle set is generally not Gaussian, we propose in this paper two methods allowing to perform a data fusion from two different clouds of particles (see Fig. 1). The first method is a transformation of the particle representation to a GMM [14]. The second algorithm approximates the particles by a Parzen representation [15].

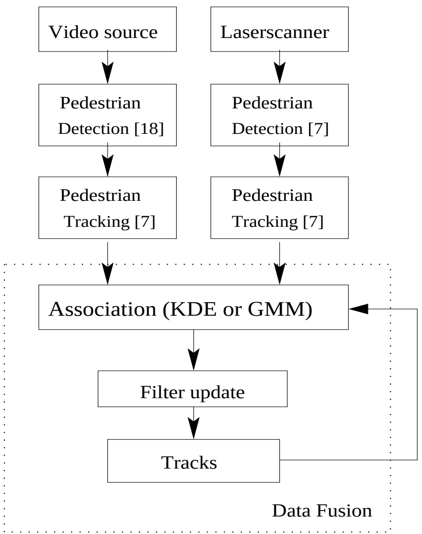

Figure 1: Data fusion architecture using lidar and vision information.

2.2 Description of the vehicle and sensors

In the CityVIP project framework, a benchmarh data set has been made using the IGN test vehicle which was equipped with an IBEO ALASCA XT. The IBEO laserscanner (see Fig. 2) has a variable scan area up to 270^o but limited here to 180^o for our experiments. The laserscanner is mounted in the center of the frontal area of test vehicle. From this position the sensor can detect all relevant objects in front of the vehicle. The manufacturer indicates that the IBEO sensor has a range measurement up to 128 m with accuracy of +/- 5 cm. The angle of resolution varies with scan frequency (at 20 Hz the resolution angle is 0.5^o) thus providing 300 measurements per channel and scan. These scan planes have a total opening angle of approx. 3.2^o. One Marlin 146-C video camera on the bottom of the sensor laser (see Fig. 2) simultaneously records the scene.

Figure 2: Left, the IBEO ALASCA XT Laserscanner and the Marlin 146-C camera. Right, the IGN test vehicle.

3 Sequential Monte Carlo Methods

In the following section, the theory of the sequential Monte Carlo methods in the framework of object tracking is briefly reminded. For more details, the reader can refer to Doucet's works [13].

3.1 General

Let us consider a discrete dynamic system:

$$\mathbf{X}_k = f(\mathbf{X}_{k-1}) + \mathbf{W}_k \tag{1}$$

$$\mathbf{Z}_k = h(\mathbf{X}_k) + \mathbf{V}_k \tag{2}$$

where $\mathbf{X}_k$ represents the state vector at instant k. No assumption is made about the two functions f and h, whereas $\mathbf{Z}_k$ and $\mathbf{V}_k$ are supposed to be two independent white noises.

3.2 Particle Filters

Particle filters provide an approximate Bayesian solution to discrete time recursive problems by updating an approximate description of the posterior filtering density $p(\mathbf{x}_k|\mathbf{z}_{1:k})$. This posterior density function represents some degree of belief in the state $\mathbf{x}_k$ at time k, given the data $\mathbf{z}_{1:k}$ up to time k.

The main purpose of particle filters is to approximate the *a priori* distribution of the recursive Bayesian filter

$p(\mathbf{x}_k|\mathbf{z}_{1:k-1})$ as a set of N samples, using the following equation:

$$p(\mathbf{x}_k|\mathbf{z}_{1:k-1}) = \frac{1}{N}\sum_{i=1}^{N}\delta(\mathbf{x}_k - \mathbf{x}_k^i) \quad (3)$$

where δ is the discrete Dirac function. Then the *a posteriori* distribution $p(x_k|z_{1:k})$ can be estimated by:

$$p(\mathbf{x}_k|\mathbf{z}_{1:k}) = p(\mathbf{z}_k|\mathbf{x}_k)\sum_{i=1}^{N} p(\mathbf{x}_k|\mathbf{x}_{k-1}^i) \quad (4)$$

This approach can be implemented using a bootstrap filter or a Sampling Importance Resampling Particle Filter (SIR PF).

4 Decentralized Particle Fusion Algorithms

In the case of particle filtering, data association is a crucial problem for a better resampling of particle set in order to keep all prior distribution information (for example multi-modalities). In fact, one of the main weakness of particle filters is the inability to adequately explore the state space if the support of the prior distribution has little overlap with the likelihood function. This problem can occur if a measurement is an outlier, if the likelihood function is highly peaked, or if the process noise is small. A solution is to fit kernels or mixture models to the samples, which can take into account more efficiently the unspecified feature of the particle distribution predicted by the SIR filter. Mixture models and kernel methods are used in this paper in order to improve the Bayesian Decentralized Data Fusion (BDDF).

So, we propose to introduce the association matrix $\mathbf{A}_k$ to describe the association between the particle set (representing measurements) $\mathbf{Z}_k$ and the particle set (representing targets) $\mathbf{X}_k$. We propose to estimate $\mathbf{A}_k$ with either a parametric method like GMM or non-parametric method like KDE. Finally, we also define a Bernoulli random variable $w_h \in \{w_1, w_2\}$ given by $w_h = w_1$ if the associated event is classified as fused data or $w_h = w_2$ in all other cases.

4.1 GMM approximation for Fusion computation

In order to better approximate the particle set distribution, allowing a smart data fusion, we propose, to approximate the estimated distribution with a Gaussian Mixture Model (GMM). In a GMM model, the probability distribution of a particle set $\mathbf{x}$ is a mixture of M Gaussian probability density functions (GPDF), defined as follows:

$$p(\mathbf{x}|\Theta) = \sum_{m=1}^{M}\alpha_m p(\mathbf{x}|\theta_m) \quad (5)$$

where $\theta_1, \ldots\theta_M$ are the parameters of the Gaussian distributions and $\alpha = [\alpha_1, \ldots, \alpha_M]$ is the weighted vector, such that $\sum_{m=1}^{M}\alpha_m = 1$. The complete set of parameters that specify the mixture model is $\Theta = (\alpha; \theta_1, \ldots, \theta_M)$, with each parameter $\theta_m = (\mu_m, \Sigma_m)$ consisting of a mean vector μ and a covariance matrix Σ. Considering a 2-dimensional feature-vector, $\mathbf{X} = \{\mathbf{x}_k^{1,l}, \ldots, \mathbf{x}_k^{N_x,l}\}_{l=1}^{N}$ denotes the vector composed by several particle sets and w_h the likelihoods of each class which are described as linear combinations of Gaussian mixture probability density functions:

$$p(\mathbf{X}|w_h, \Theta^{i,j}) = \sum_{m=1}^{M}\alpha_m^{i,j} p(\mathbf{X}|\theta_m^{i,j}) \quad (6)$$

where each GPDF component is given by

$$p(\mathbf{X}|\Theta_m^{i,j}) = \frac{1}{\sqrt{(2\pi)|\Sigma_m^{i,j}|}} \cdot \exp\left[-\frac{1}{2}(\mathbf{X}-\mu_m^{i,j})^t(\Sigma_m^{i,j})^{-1}(\mathbf{X}-\mu_m^{i,j})\right] \quad (7)$$

The GMM parameters for each object class are estimated using the expectation-maximization (EM) algorithm. The EM algorithm estimates the maximum likelihood parameters in statistical models with variables that are not observed, given initial moments. For more details, a book [16] is devoted entirely to EM algorithm and its applications. The value of mixture probabilities $x_k \in w_1$ is chosen by the maximum likelihood estimator as a likely data fusion:

$$\mathbf{A}_k^{i,j}|h = \arg\max_h(\alpha_m^{i,j}) \quad (8)$$

>From this probability, a threshold β allows to accept or reject an association track/observation given by $\mathbf{A}_k^{i,j}$. If the association is validated, the next step consists in weights computing of all the points belonging to the Gaussian mixture given by (6). Thus the point list $\mathbf{L}^{i,j}$ is calculated as follows:

$$\mathbf{L}^{i,j} = p(\mathbf{X}|w_h, \Theta^{i,j}) \quad (9)$$

This algorithm is summarized in Algorithm 1.

4.2 KDE methods for Fusion computation

For Parzen density estimates, any type of kernel may be used to represent a probability distribution. However, Gaussian kernels are preferred, as most of its operations are closed form and therefore efficient. The Parzen density estimator is similar to a GMM except that each component has the same covariances. The equation for a Parzen density estimate with a Gaussian kernel is similar to the mixture of Gaussian functions which is:

$$p(\mathbf{x}) = \sum_{m=1}^{M}\gamma_m\varphi(\mathbf{x};\mu_m,\Sigma_m) \quad (10)$$

where $\varphi(\mathbf{x})$ is the Gaussian probability density on $\mathbf{x}$ and γ_m are the weights where $\sum_{m=1}^{M}\gamma_m = 1$.

We propose an approach to build a non-parametric model based on kernel functions, allowing a smart selection of the most pertinent data fusion from a likelihood analysis function. A likelihood discriminating function permits the classification of each particles as the association gravity center

Algorithm 1 BDDF with GMM approximation

1. Compute the matrix $\mathbf{A}_k$ for all measurements and objects from GMM approximation.

   if $(\mathbf{A}_k \leq \beta)$
   Compute the weights $\mathbf{w}_k^i = p(\mathbf{X}|w_h, \Theta^{i,j})$ and normalize, i.e, $\mathbf{w}_k^l = \frac{\mathbf{w}_k^l}{\sum_{l=1}^N \mathbf{w}_k^l}$.
   Generate a new set $\{\mathbf{x}_k^{i,l*}\}_{l=1}^N$ by resampling from $\{\mathbf{x}_k^{i,l}\}_{l=1}^N$, according to the probability $Pr(\mathbf{x}_k^{i,l*} = \mathbf{x}_k^{i,l}) = \mathbf{w}_k^{i,l}$.

   else
   $\mathbf{A}_k^{i,j} = 0$ then $\{\mathbf{x}_k^{i,l}\} = \{\mathbf{x}_{k-1}^{i,l}\}$.
   endif

2. Predict (simulate) new particles, i.e, $\mathbf{x}_{\mathrm{k}+1}^{\mathrm{i,l}} = \mathrm{f}(\mathbf{x}_{\mathrm{k}}^{\mathrm{i,l*}}, \mathbf{v}_{\mathrm{k}})$, $l = 1, \ldots, N$ using different noise realizations for the particles.

3. Increase k and iterate to item 2.

or not. This method is not supervised, so no prior knowledge is required to process the association.

Let $\mathbf{X} = \{\mathbf{x}_k\}_{k=1,\ldots,N_s}$ denote the vector composed by several particle sets.

The likelihood function $p(\mathbf{X}|w_h)$ allows to compute the probability that a particle belongs to a fused data. We propose to model the likelihood $p(\mathbf{X}|w_k)$ by a non-parametric model using an estimation based on kernel functions (Parzen window model).

$$p(\mathbf{X}|w_k) = \frac{1}{2 \cdot N} \sum_{l=1}^{N} \sum_{h=1}^{N} \varphi(\mathbf{x}_k^{i,l}, \mathbf{x}_k^{j,h}) \tag{11}$$

Finally $\varphi(\mathbf{x}_k^{i,l}, \mathbf{x}_k^{j,h})$ is the kernel function which allows to modify the zone of influence of a point with its neighbours, it is defined by:

$$\varphi(\mathbf{x}_k^{i,l}, \mathbf{x}_k^{j,h}) = \exp[-\lambda_c \cdot d_c(\mathbf{x}_k^{i,l}, \mathbf{x}_k^{j,h})] \tag{12}$$

The λ_c parameter permits to adjust the weights. The d_c distance used is a Mahalanobis distance defined by:

$$d_c(\mathbf{x}_k^{i,l}, \mathbf{x}_k^{j,h}) = (\mathbf{x}_k^{i,l} - \mathbf{x}_k^{j,h}) {\Sigma_\varphi^{i,j}}^{-1} (\mathbf{x}_k^{i,l} - \mathbf{x}_k^{j,h})^T \tag{13}$$

with Σ_φ^i and Σ_φ^j, the covariance matrix given by the tracking algorithms representing the uncertainties on pedestrian position.

$$\Sigma_\varphi^{i,j} = \Sigma_\varphi^i + \Sigma_\varphi^j \tag{14}$$

The 2D particle having the highest probability $\mathbf{x}_k \in w_1$ is chosen by the maximum likelihood estimator as a likely data fusion:

$$A_k^{i,j}|h = \arg\max_h (p(\mathbf{X}|w_h \in w_1)) \tag{15}$$

>From this probability, a threshold δ allows to accept or reject a track/observation association given by $A_k^{i,j}$.

If the data association is validated, the next step consists in weights computing of all the points belonging to the association gravity center given by (15). Thus the point list $L^{i,j}$ is calculated as follows:

$$L^{i,j} = \varphi(\mathbf{x}_k^{i,l}, \mathbf{x}_k^{j,h}) \tag{16}$$

This algorithm is summarized in Algorithm 2.

Algorithm 2 BDDF with KDE methods

1. Compute the matrix $\mathbf{A}_k$ for all measurements and objects from KDE methods.

   if $(\mathbf{A}_k \leq \delta)$
   Compute the weights $\mathbf{w}_k^i = \varphi(\mathbf{x}_h^j, \mathbf{x}_l^i)$ and normalize, i.e, $\mathbf{w}_k^l = \frac{\mathbf{w}_k^l}{\sum_{l=1}^N w_k^l}$.
   Generate a new set $\{\mathbf{x}_k^{i,l*}\}_{l=1}^N$ by resampling from $\{\mathbf{x}_k^{i,l}\}_{l=1}^N$, according to the probability $Pr(\mathbf{x}_k^{i,l*} = \mathbf{x}_k^{i,l}) = \mathbf{w}_k^{i,l}$.

   else
   $\mathbf{A}_k^{i,j} = 0$ then $\{\mathbf{x}_k^{i,l}\} = \{\mathbf{x}_{k-1}^{i,l}\}$.
   endif

2. Predict (simulate) new particles, i.e, $\mathbf{x}_{\mathrm{k}+1}^{\mathrm{i,l}} = \mathrm{f}(\mathbf{x}_{\mathrm{k}}^{\mathrm{i,l*}}, \mathbf{v}_{\mathrm{k}})$, $l = 1, \ldots, N$ using different noise realizations for the particles.

3. Increase k and iterate to item 2.

5 Experiments

This section presents simulations and experiments which have allowed to validate the two BDDF algorithms based on GMM or KDE methods.

5.1 Simulations

Simulations have been conducted to compare the performance of target tracking using BDDF with GMM approximation and BDDF with KDE methods. First, we propose a study on approximation of the particle by GMM and KDE methods. Fig. 3(a/b) shows a particle sample set of 1000 particles which represent two clouds of particles from two different sensors. Fig. 3(c/d) shows a data fusion with a Gaussian mixture model using EM algorithm while Fig. 3(e/f) shows the equivalent data fusion via KDE methods. From these figures, one can see that using the GMM algorithm (See Fig. 3(c/d)) a better approximation of the particle set is obtained.

Secondly, we propose a study on the mean square error on radial position where BDDF with GMM approximation and BDDF with KDE methods have been repeated 100 times.

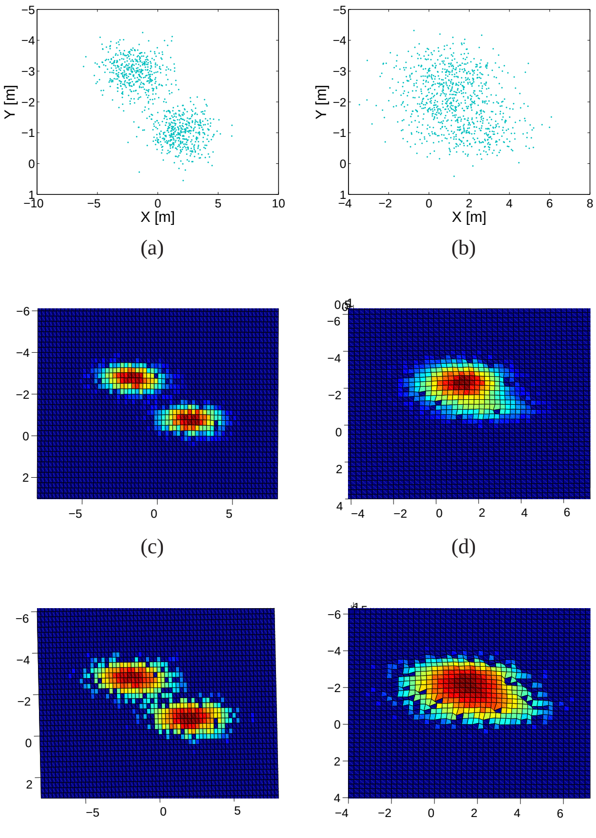

Figure 3: Examples of approximation of probability density function given by in left two separate clouds of particles and in right two mix clouds of particles.

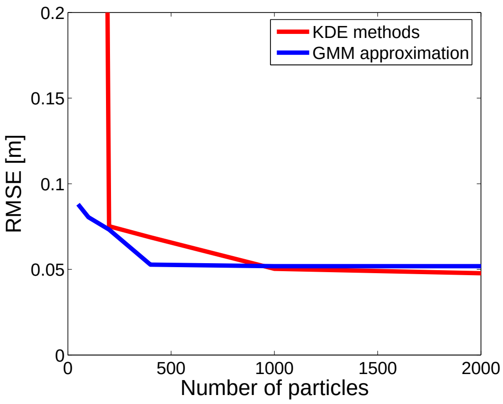

Figure 4: Comparison of Root Mean Square Errors on radial position between GMM and KDE methods while particle number increases.

In Fig. 4, it can be seen that GMM methods are better than KDE methods when less 1000 particles are used. But when more 1000 particles are used, KDE methods become better then GMM methods. So, before 1000 particles, GMM methods are more accurate, but when the number of particles increases the KDE methods become better. Fig. 5 shows the number of particles increases, the computing cost as much as the reliability increase with KDE methods while this point is not true with GMM methods because the computing cost stays the same whatever the number of particles [18]. This last point is very interesting, because the processing time is of prime importance in automotive application.

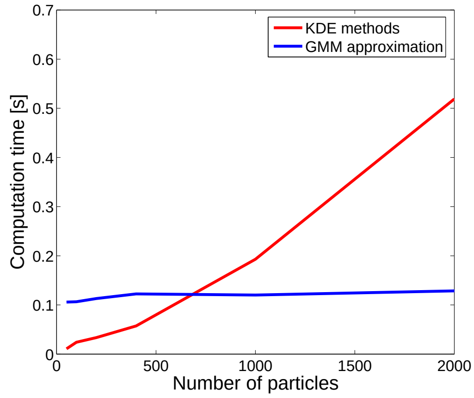

Figure 5: Comparison of the computation time between GMM and KDE methods (in the case of two sensors) while the particle number increases.

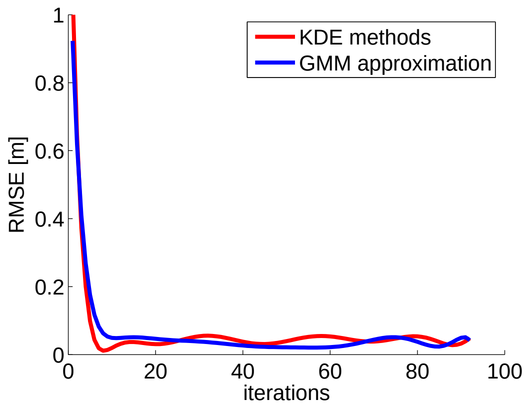

Figure 6: Comparison of Root Mean Square Errors on radial position between GMM and KDE methods during a pedestrian tracking presented in Fig. 5.

5.2 Experiments on real data

We present here various results of laser and video data fusion. Lidar and camera data are not given in the same reference frame. Thus, we choose the reference frame related to the lidar for fusion. The SIR PF with Parzen Window association was tested on real data in many different situations provided by Renault, the French vehicle manufacturer (see Section 2). The presented scenario (see Fig. 7) including several pedestrians (> 5) who appear and disappear in the sensor area representing different situations such as an urban scene, a semi-urban scene, or a car park. Of course, the pedestrians move in all directions. The vehicle moves

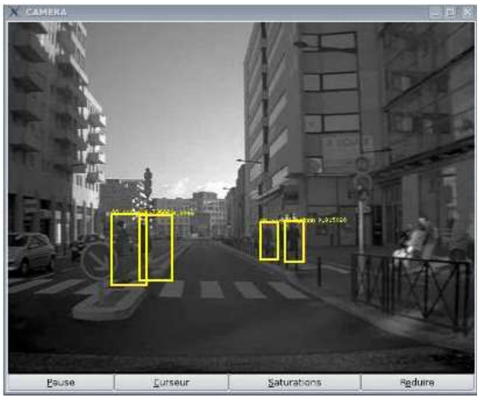

Figure 7: Detection example in a cross section after a centralized fusion from lidar and image data. Pedestrian in various orientations are detected.

at a speed ranging from 0 to 50 km/h, which allows to test the robustness of this method. Generally, in pedestrian classification framework, lidar measurements can generate false track [6]. These lidar measurements result in general from fixed objects suitable for an urban environment. False lidar measurements can be assimilated with security barriers, poles, trees, etc. The fusion with a camera sensor allows to remove false tracks provided by the lidar. Finally, Fig. 6 confirms a better approximation of GMM methods during a pedestrian tracking (Fig. 5) with 400 particles used (Fig. 4).

6 Conclusions

In this article, we have presented two methods for performing consistent and efficient Bayesian Decentralized Data Fusion for particle filtering. This work takes place in the LOVe project framework. In order to track more easily the pedestrian random movements which can include abrupt trajectory changes, a SIR PF has been chosen. We examined the relevance of transforming the particle set to either GMM or KDE estimates for data fusion. To summarize, the sample sets with KDE methods are more accurate than GMM approximation when the number of particles increases (over 1000 particles). However, when the number of particles decreases, GMM method becomes better. The experimental results on real data including more than 10 different scenarios recorded aboard Renault vehicle in real car traffic demonstrate the effectiveness of these two methods.

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